

## Derivation and validation of a generalized Darcy law for Herschel-Bulkley fluids in porous media

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**Abstract.** Large resources of heavy and highly viscous crude oil exhibit non-Newtonian behavior that cannot be represented adequately by the classical Darcy law. A particularly important feature of many structured oils is the existence of a yield stress and, consequently, a threshold pressure gradient below which macroscopic filtration does not occur. This study develops an analytical generalized Darcy relation for a Herschel-Bulkley fluid flowing through a homogeneous porous medium. The derivation starts from the laminar tube-flow solution and introduces porosity, an equivalent hydraulic radius, and a pore-channel tortuosity coefficient to obtain a porous-medium velocity equation. The resulting relation is nonlinear and explicitly includes yield stress, consistency index, flow-behavior index, permeability, and porosity. Its limiting forms are examined for power-law, Newtonian, and Bingham fluids. The analytical equation is then compared with a widely used simplified semi-empirical relation for permeability values of  $1.0 \times 10^{-13}$ ,  $2.0 \times 10^{-13}$ , and  $3.0 \times 10^{-13}$  m<sup>2</sup> and flow-behavior indices of 0.90, 1.05, and 1.20. The two formulations show similar trends over the investigated engineering range, although the deviation becomes more noticeable near the onset of flow and for stronger departures from Newtonian behavior. An explicit expression for the threshold pressure gradient is also obtained, eliminating the need to treat the corresponding coefficient as purely empirical. The proposed formulation can support screening calculations and reservoir-flow models involving yield-stress oils and other Herschel-Bulkley fluids.

**Keywords:** Herschel-Bulkley model; generalized Darcy law; non-Newtonian fluid; porous-medium flow; threshold pressure gradient; heavy oil.

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**1. Introduction.** The progressive depletion of conventional petroleum resources has increased interest in heavy oils, bitumen, and other unconventional hydrocarbons. These resources represent a substantial part of the remaining hydrocarbon base, but their development is complicated by high viscosity, complex composition, and non-Newtonian flow behavior [1-4].

High viscosity reduces well deliverability and may make production uneconomic unless the fluid mobility is improved. Thermal enhanced-oil-recovery methods are frequently applied because viscosity generally decreases strongly with increasing temperature [5, 6]. Nevertheless, reliable reservoir calculations still require a constitutive flow relation that remains valid before, during, and after thermal or chemical treatment.

The rheology of structured oils is governed by the formation, deformation, and breakdown of internal networks associated with asphaltenes, resins, waxes, and dispersed phases. These effects have been discussed in rheological studies of anomalous oils [7], reviews of enhanced recovery from non-Newtonian reservoirs [8], and experimental investigations of threshold pressure gradients in porous media [9].

Accounting for non-Newtonian behavior is essential in field-development forecasting. A model based on the classical Darcy law may overestimate mobility, drainage radius, and ultimate recovery when a fluid possesses a yield stress. In such systems, the local pressure gradient must exceed a finite threshold before continuous flow begins. Regions in which this condition is not

satisfied become stagnant or weakly mobilized zones, particularly far from producing wells. Neglecting these zones in reservoir simulation can produce optimistic recovery predictions [10].

For a Newtonian fluid, shear stress is proportional to shear rate:

$$\tau = \mu \dot{\gamma}. \quad (1)$$

where  $\tau$  is shear stress,  $\mu$  is dynamic viscosity, and  $\dot{\gamma}$  is shear rate. Fluids that do not satisfy this linear relation are classified as non-Newtonian. The Bingham, Ostwald-de Waele (power-law), and Herschel-Bulkley models are commonly written as follows [11]:

$$\tau = \tau_0 + \mu_B \dot{\gamma}, \quad (2)$$

$$\tau = K \dot{\gamma}^n, \quad (3)$$

$$\tau = \tau_0 + K \dot{\gamma}^n. \quad (4)$$

Here  $\tau_0$  is the yield stress,  $\mu_B$  is the Bingham plastic viscosity,  $K$  is the consistency index, and  $n$  is the flow-behavior index. The Herschel-Bulkley equation is the most general of these three expressions: it reduces to the power-law model when  $\tau_0 = 0$ , to the Bingham model when  $n = 1$ , and to the Newtonian model when  $\tau_0 = 0$  and  $n = 1$ .

Porous-medium flow of Bingham fluids has been treated analytically and numerically [12], whereas power-law filtration has received broader attention through packed-bed, pore-network, and continuum formulations [13, 14]. The yield-stress term makes Herschel-Bulkley flow substantially more difficult, especially under transient conditions, because the mobile region itself depends on the pressure field.

The Herschel-Bulkley model is widely used for drilling fluids, polymer solutions, emulsions, and structured crude oils because it captures both a finite yield stress and nonlinear post-yield behavior [15, 16]. Pipe-flow formulations and explicit approximations for pressure loss have been developed for operational calculations [17-19].

Chevalier et al. [20] experimentally investigated yield-stress-fluid flow through beds of glass beads with diameters from 0.26 to 2 mm. A Carbopol solution and a water-in-oil emulsion were both described satisfactorily by the Herschel-Bulkley model. Reported parameter ranges included  $\tau_0 = 8\text{-}89$  Pa and  $K = 3.6\text{-}49$  Pa $\cdot$ s $^n$  for the Carbopol system, and  $\tau_0 = 54\text{-}71$  Pa and  $K = 12.7\text{-}13.4$  Pa $\cdot$ s $^n$  for the emulsion, with  $n \approx 0.36$ . Related rheological behavior has also been reported for emulsions and asphaltene-rich oils [21, 22].

The objective of the present work is to derive a generalized Darcy law for Herschel-Bulkley fluids from the laminar capillary-flow solution, evaluate its limiting cases, obtain an analytical threshold-gradient condition, and compare the result with a simplified semi-empirical relation used in engineering practice.

**2. Methodological part.** The derivation is based on steady, laminar, single-phase flow of an incompressible Herschel-Bulkley fluid through a homogeneous and isotropic porous medium. Fluid properties, porosity, and permeability are assumed constant over the representative elementary volume. Inertial effects, compressibility, gravity segregation, and time-dependent rheology are neglected. The pressure coordinate is selected so that  $G = -dP/dz > 0$  in the direction of flow.

### Porous-medium flow relations

The classical Darcy equation relates superficial velocity to pressure gradient through permeability and viscosity:

$$v = -\frac{k}{\mu} \frac{dP}{dx}. \quad (5)$$

For a packed bed, the Blake-Kozeny relation can be written as

$$v = -\frac{D_p^2 \phi^3}{150 \mu (1-\phi)^2} \frac{dP}{dx}. \quad (6)$$

and comparison with Darcy's law gives

$$k = \frac{D_p^2 \phi^3}{150 (1-\phi)^2}. \quad (7)$$

A more general pore-geometry representation introduces a coefficient  $C$  that accounts for tortuosity and deviations from an ideal bundle of straight capillaries [23]:

$$k = \frac{D_p^2 \phi^3}{72C(1-\phi)^2}. \quad (8)$$

At low velocities, viscous forces dominate. At higher velocities, the inertial contribution represented by the Ergun equation may become important [24]:

$$\frac{dP}{dx} = \frac{150\mu v(1-\phi)^2}{D_p^2 \phi^3} + \frac{1.75\rho v^2(1-\phi)}{D_p \phi^3}. \quad (9)$$

Only the viscous regime is considered below; therefore, the second term in Eq. (9) is omitted in the derivation.

### Simplified semi-empirical relation

A frequently used generalized Darcy relation for Herschel-Bulkley fluids is [25]

$$v = \left[ \frac{k}{\mu_{ef}} \left( -\frac{\partial P}{\partial x} - \alpha_0 \right) \right]^{\frac{1}{n}}, \quad (10)$$

where the mobility factor is

$$\frac{k}{\mu_{ef}} = \frac{1}{2K} \left( \frac{n\phi}{1+3n} \right)^n \left( \frac{8k}{\phi} \right)^{\frac{n+1}{2}}, \quad (11)$$

and the threshold pressure gradient is commonly represented as

$$\alpha_0 = \frac{\beta \tau_0}{\sqrt{k}}, \quad (12)$$

The coefficient  $\beta$  is usually established experimentally. A corresponding expression for a Bingham fluid was proposed in the form [26]

$$v = \frac{k}{\mu_B} \left( -\frac{\partial P}{\partial x} - \alpha_0 \right). \quad (13)$$

In Eqs. (10) and (13), the subtraction of  $\alpha_0$  from the applied gradient introduces the onset-of-flow condition, but the coefficient itself is not obtained directly from the constitutive equation.

For a power-law fluid with zero yield stress, Christopher and Middleman [27] derived the analytical packed-bed relation

$$v = \left( \frac{k}{\mu_{ef}} \frac{dP}{dx} \right)^{\frac{1}{n}}, \quad (14)$$

with the effective viscosity

$$\mu_{ef} = \frac{K}{12} \left( 9 + \frac{3}{n} \right)^n (150k\phi)^{\frac{1-n}{2}}. \quad (15)$$

### Derivation of the generalized Darcy law

The volumetric flow rate of a Herschel-Bulkley fluid in a circular tube of radius  $R$  can be expressed as [15, 16]

$$Q = \frac{\pi n}{K^{\frac{1}{n}}} \frac{\left( -\frac{RdP}{2dz} - \tau_0 \right)^{1+\frac{1}{n}}}{\left( -\frac{1dP}{2dz} \right)^3} \left( \frac{\left( -\frac{RdP}{2dz} - \tau_0 \right)^2}{3n+1} + \frac{2\tau_0 \left( -\frac{RdP}{2dz} - \tau_0 \right)}{2n+1} + \frac{\tau_0^2}{n+1} \right). \quad (16)$$

Dividing Eq. (16) by the tube cross-sectional area gives the mean velocity

$$v = \frac{n}{K^{\frac{1}{n}} R^2} \frac{\left( -\frac{RdP}{2dz} - \tau_0 \right)^{1+\frac{1}{n}}}{\left( -\frac{1dP}{2dz} \right)^3} \left( \frac{\left( -\frac{RdP}{2dz} - \tau_0 \right)^2}{3n+1} + \frac{2\tau_0 \left( -\frac{RdP}{2dz} - \tau_0 \right)}{2n+1} + \frac{\tau_0^2}{n+1} \right). \quad (17)$$

To transfer Eq. (17) from a single capillary to a porous medium, three geometric corrections are introduced. First, the velocity is multiplied by porosity  $\phi$  to obtain a superficial, or Darcy, velocity. Second, the pressure gradient is divided by  $C$  to account for pore-path tortuosity. Third, the tube radius is replaced by an equivalent pore radius. Since the hydraulic radius of a circular tube is  $r_h = R/2$  and the mean hydraulic radius of a packed bed is  $r_h = D_p \phi / [6(1-\phi)]$ , the equivalent radius is  $R = D_p \phi / [3(1-\phi)]$ . Combining this result with Eq. (8) gives  $R = \sqrt{\frac{8kC}{\phi}}$ .

After these substitutions, Eq. (17) becomes

$$v = \frac{\phi n}{K^{\frac{1}{n}} \left( \sqrt{\frac{8kC}{\phi}} \right)^2} \frac{\left( -\sqrt{\frac{8kC}{\phi}} \frac{dP}{2C dz} - \tau_0 \right)^{1+\frac{1}{n}}}{\left( -\frac{1}{2C} \frac{dP}{dz} \right)^3} \left( \frac{\left( -\sqrt{\frac{8kC}{\phi}} \frac{dP}{2C dz} - \tau_0 \right)^2}{3n+1} + \frac{2\tau_0 \left( -\sqrt{\frac{8kC}{\phi}} \frac{dP}{2C dz} - \tau_0 \right)}{2n+1} + \frac{\tau_0^2}{n+1} \right). \quad (18)$$

Algebraic simplification yields the generalized Darcy relation

$$v = \frac{\phi^2 n}{8K^{\frac{1}{n}} k C} \frac{\left( -\sqrt{\frac{2k}{\phi C}} \frac{dP}{dz} - \tau_0 \right)^{1+\frac{1}{n}}}{\left( -\frac{1}{2C} \frac{dP}{dz} \right)^3} \left( \frac{\left( -\sqrt{\frac{2k}{\phi C}} \frac{dP}{dz} - \tau_0 \right)^2}{3n+1} + \frac{2\tau_0 \left( -\sqrt{\frac{2k}{\phi C}} \frac{dP}{dz} - \tau_0 \right)}{2n+1} + \frac{\tau_0^2}{n+1} \right). \quad (19)$$

Equation (19) is the principal analytical result of this study. It explicitly links the superficial velocity to the applied pressure gradient, permeability, porosity, pore-geometry coefficient, and all three Herschel-Bulkley rheological parameters.

### Limiting cases and flow-onset condition

When  $\tau_0 = 0$  and  $n \neq 1$ , Eq. (19) reduces to the generalized power-law form

$$v = \frac{\phi^2 C^2 n}{(3n+1)kK^{\frac{1}{n}}} \left( \frac{2k}{\phi C} \right)^{\frac{3n+1}{2n}} \left( -\frac{dP}{dz} \right)^{\frac{1}{n}}. \quad (20)$$

For  $C = 25/12$ , Eq. (20) is consistent with the packed-bed expression in Eq. (14). Setting  $n = 1$  additionally recovers the classical Darcy equation

$$v = -\frac{k}{K} \frac{dP}{dz}. \quad (21)$$

where  $K$  becomes the Newtonian viscosity. If  $n = 1$  while  $\tau_0 \neq 0$ , the Herschel-Bulkley model reduces to the Bingham model, and Eq. (19) gives

$$v = \frac{\phi^2 C^2}{kK} \frac{1}{\left( -\frac{dP}{dz} \right)^3} \left( \frac{\left( -\sqrt{\frac{2k}{\phi C}} \frac{dP}{dz} - \tau_0 \right)^4}{4} + \frac{2\tau_0 \left( -\sqrt{\frac{2k}{\phi C}} \frac{dP}{dz} - \tau_0 \right)^3}{3} + \frac{\tau_0^2 \left( -\sqrt{\frac{2k}{\phi C}} \frac{dP}{dz} - \tau_0 \right)^2}{2} \right). \quad (22)$$

which can be rearranged as

$$v = \frac{k}{K} \left( -\frac{dP}{dz} - \frac{2}{3} \sqrt{\frac{2\phi C}{k}} \tau_0 + \frac{\phi^2 C^2}{12} \tau_0^4 \frac{1}{\left( -\frac{dP}{dz} \right)^3} \right). \quad (23)$$

A necessary condition for nonzero flow follows directly from the requirement that the yielded annular region exist:

$$-\frac{dP}{dz} > \sqrt{\frac{\phi C}{2k}} \tau_0 \approx \sqrt{\frac{\phi}{k}} \tau_0. \quad (24)$$

Accordingly, the threshold pressure gradient is

$$\alpha_0 = \sqrt{\frac{\phi C}{2k}} \tau_0 \approx \sqrt{\frac{\phi}{k}} \tau_0. \quad (25)$$

Comparison of Eqs. (12) and (25) gives an analytical estimate of the empirical coefficient

$$\beta = \sqrt{\frac{\phi C}{2}} \approx \sqrt{\phi}. \quad (26)$$

Thus,  $\beta$  is related primarily to porosity and the pore-geometry coefficient rather than being an unrestricted fitting parameter.

**3. Results and discussion.** The analytical relation in Eq. (19) was compared with the simplified semi-empirical equation, Eq. (10). The parameter set used for the calculations is summarized in Table 1. The selected permeability values represent low-permeability porous media, while the three flow-behavior indices cover mildly shear-thinning, near-linear, and shear-thickening post-yield responses.

Table 1

| Parameter                      | Value                                                                             |
|--------------------------------|-----------------------------------------------------------------------------------|
| Yield stress, $\tau_0$         | 0.3 Pa                                                                            |
| Consistency index, $K$         | $5 \times 10^{-3} \text{ Pa} \cdot \text{s}^n$                                    |
| Pore-geometry coefficient, $C$ | 25/12                                                                             |
| Porosity, $\phi$               | 0.10                                                                              |
| Flow-behavior index, $n$       | 0.90; 1.05; 1.20                                                                  |
| Permeability, $k$              | $1.0 \times 10^{-13}$ ; $2.0 \times 10^{-13}$ ; $3.0 \times 10^{-13} \text{ m}^2$ |

Figure 1 presents the velocity-pressure-gradient curves for  $k = 1.0 \times 10^{-13} \text{ m}^2$ . The solid curves correspond to Eq. (19), whereas the black dashed curves represent Eq. (10). The results confirm that the apparent mobility is highly sensitive to  $n$ . Over the plotted range, changing  $n$  from 0.90 to 1.20 alters the predicted velocity by several times, even though all other properties remain fixed.

The simplified and analytical equations produce the same overall ordering of the curves and comparable engineering trends. Their agreement is closest for the larger value of  $n$  and decreases as the rheology departs more strongly from the near-linear case. The largest relative differences occur close to the threshold gradient, where both predicted velocities approach zero and small absolute changes produce large percentage deviations. Therefore, Eq. (10) is suitable for preliminary calculations within the investigated range, whereas Eq. (19) is preferable when the onset of flow or the detailed nonlinear response is important.

The dotted reference curves illustrate the response obtained when yield-stress and nonlinear effects are omitted. Their departure from the Herschel-Bulkley solutions demonstrates that the classical Darcy law cannot represent the combined influence of a threshold gradient and a non-unit flow-behavior index.

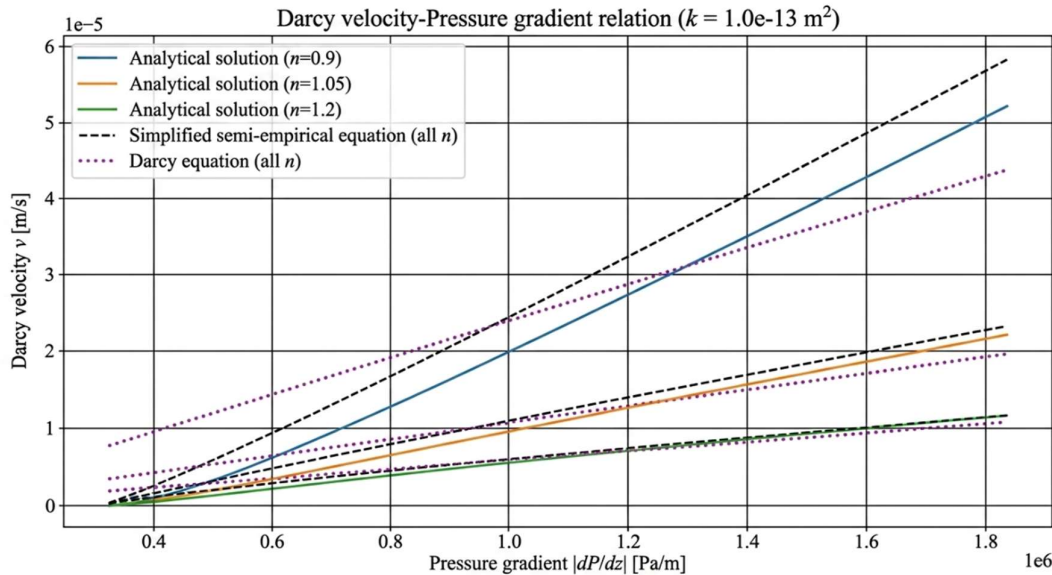


Figure 1. Darcy velocity as a function of pressure gradient for  $k = 1.0 \times 10^{-13} \text{ m}^2$ , calculated using Eqs. (10) and (19)

Figures 2 and 3 show the corresponding results for permeabilities of  $2.0 \times 10^{-13}$  and  $3.0 \times 10^{-13} \text{ m}^2$ . Increasing permeability raises the Darcy velocity and shifts the flow-onset gradient to lower values, in agreement with Eq. (25), which gives  $\alpha_0 \propto 1/\sqrt{k}$ . Thus, a more permeable medium not only provides greater mobility after yielding but also requires a smaller pressure gradient to initiate filtration.

The consistent pattern across the three permeability levels supports the internal correctness of the derived scaling. At the same time, the comparison should not be interpreted as universal validation for all porous structures and rheological states. The derivation treats the porous medium through an equivalent capillary radius and a single geometric coefficient  $C$ ; experimental calibration remains advisable for highly heterogeneous rocks, broad pore-size distributions, thixotropic fluids, or regimes in which inertial effects become significant.

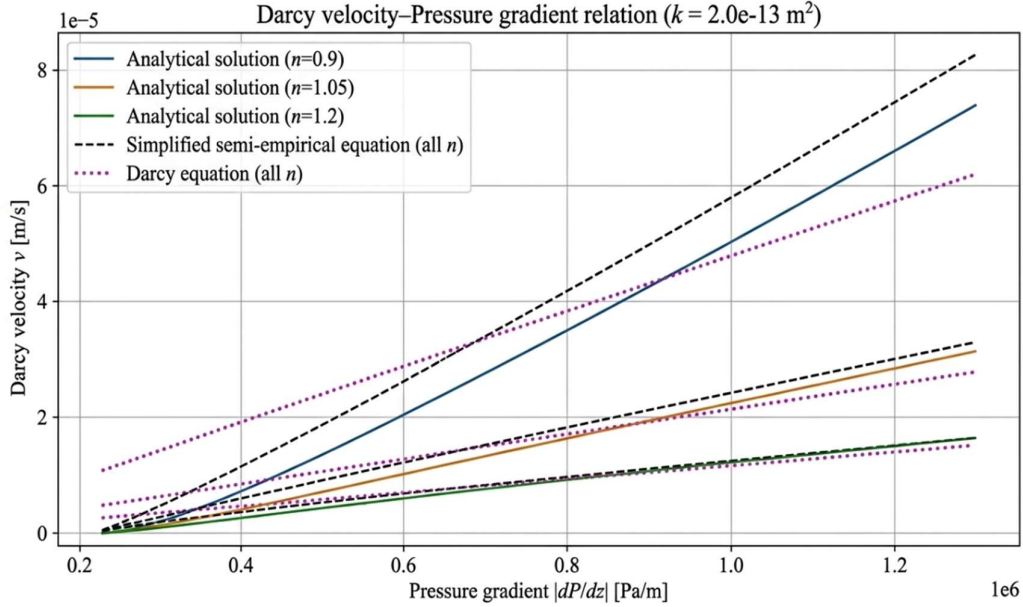


Figure 2. Darcy velocity as a function of pressure gradient for  $k = 2.0 \times 10^{-13} \text{ m}^2$ , calculated using Eqs. (10) and (19)

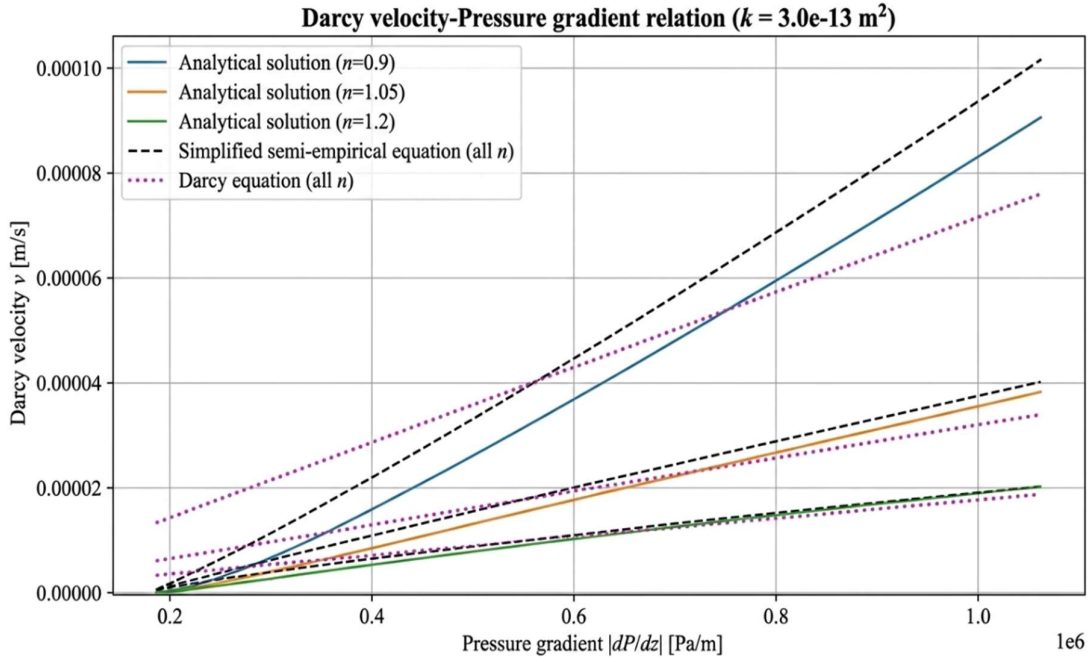


Figure 3. Darcy velocity as a function of pressure gradient for  $k = 3.0 \times 10^{-13} \text{ m}^2$ , calculated using Eqs. (10) and (19)

**4. Conclusions.** An analytical generalized Darcy law has been derived for steady laminar flow of a Herschel-Bulkley fluid through a homogeneous porous medium. The main findings are summarized below.

1. The resulting equation includes permeability, porosity, a pore-geometry coefficient, yield stress, consistency index, and flow-behavior index in a single nonlinear relation between Darcy velocity and pressure gradient.

2. The formulation correctly reduces to the generalized power-law expression, the classical Newtonian Darcy law, and a Bingham-fluid relation under the corresponding limiting assumptions.

3. A closed-form condition for the onset of filtration was obtained. The threshold gradient is proportional to yield stress and porosity and inversely proportional to the square root of permeability.

4. Comparison with a simplified semi-empirical equation showed similar engineering trends for  $k = 1.0 \times 10^{-13} - 3.0 \times 10^{-13} \text{ m}^2$  and  $n = 0.90 - 1.20$ . The simplified equation can be used for screening calculations in this range, but the analytical relation is more informative near the flow threshold and for strongly nonlinear rheology.

5. The proposed equation provides a physically interpretable basis for estimating stagnant zones and mobility in reservoirs containing structured heavy oils. Further validation should include core-scale experiments and extension to heterogeneous, transient, and temperature-dependent conditions.

#### **Conflict of interest.**

The authors declare that they have no conflict of interest in relation to this research.

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