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Choice of method for determining pressure losses during multiphase flow in vertical wells

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Abstract. This article examines the selection of a method for determining pressure losses during multiphase flow in vertical wells. An analysis of methods used to determine pressure losses during vertical multiphase flow (Hagedorn-Brown, Beggs-Brill, Orkishevsky) is conducted for parameters that vary during well operation. It is demonstrated that the Hagedorn-Brown method provides the best results for small pipe diameters (3.5 inches) and low gas-oil ratios (611–938 SCF/STB), while the Orkishevsky method is preferable for larger pipe diameters (4.5 inches) and high gas-oil ratios (938–1898 SCF/STB).
Keywords: multiphase flow; pressure losses; pipe diameter; gas-oil ratio.

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1. Introduction. When solving many practical problems of oil and gas production related to assessing the potential and optimizing the productivity of oil and gas wells (calculating bottomhole pressure, interpreting the results of hydrodynamic studies, calculating the optimal flow rate of a well, taking into account the joint operation of the formation and the lift (well-formation system), etc.), it is necessary to take into account the hydraulic characteristics of the flow of multiphase fluid flow in the well [1–3].

To solve the above problems, the use of modern mathematical methods for calculating the characteristics of multiphase flow in a wellbore is required [4–5].

The article discusses the issues of choosing a method for determining pressure losses during multiphase flow in vertical wells.

A comparative analysis of the most commonly used dependencies for determining pressure losses during the movement of multiphase flow in a well was carried out (Hagedorn and Brown [6], Orkiszewski [7] and Beggs and Brill [8] methods) for different values of input parameters. The values calculated by these methods were compared with the measured (actual) pressure loss values in the wells.

The features of calculating pressure losses using the listed methods are discussed below.

Hagedorn–Brown method

2. Methodological part. The Hagedorn–Brown equation for the pressure gradient is as follows:

$$\frac{dP}{dL}=\frac{f\rho_{mn}^2 v_m^2}{2\rho_m d}+\rho_m g+\frac{\rho_m \Delta\left(v_m^2\right)}{2dL} \qquad (1)$$

where ρ_m is the density of the gas-liquid mixture; ρ_{mn} – density of the mixture without slipping; v_m – speed of the gas-liquid mixture; d – pipe diameter; g – acceleration of gravity; f – friction coefficient.

The speed of the gas-liquid mixture is determined from the equation $v_m = \frac{q_L + q_G}{A}$, where q_L, q_G is the volumetric flow rate of liquid and gas; A – cross-sectional area of the pipe.

The density of the mixture without slipping is determined from the equation $\rho_{mn} = \rho_L \lambda_L + \rho_G(1-\lambda_L)$, where $\lambda_L = \frac{q_L}{q_L + q_G}$ is the fraction of liquid during non-slip flow of the gas-liquid mixture; ρ_L – liquid density; ρ_G – gas density.

To determine the volume fraction of the liquid phase in the tubing (liquid holdup), it is necessary to determine an auxiliary correction factor and a specified value of the liquid viscosity number CN_L. These coefficients are determined from the dependencies proposed by Duns and Ros [9], using the following dimensionless parameters:

– liquid velocity number

$$N_{LV} = v_{SL} \sqrt[4]{\frac{\rho_L}{g\sigma}}\,; \tag{2}$$

– gas velocity number

$$N_{GV} = v_{SG} \sqrt[4]{\frac{\rho_L}{g\sigma}}\,; \tag{3}$$

– pipe diameter number

$$N_D = d \sqrt[4]{\frac{\rho_L g}{\sigma}}\,; \tag{4}$$

– liquid viscosity number

$$N_L = \mu_L \sqrt[4]{\frac{g}{\rho_L \sigma^3}}\,. \tag{5}$$

Here $v_{SL} = \frac{q_L}{A}$ is the superficial velocity of the liquid, $v_{SG} = \frac{q_G}{A}$ is the superficial velocity of the gas, ρ_L is the density of the liquid, μ_L is the viscosity of the liquid, σ is the surface tension.

For the actual average speed of movement of the gas-liquid mixture, we can write

$$v_m = \frac{q_L + q_G}{A} = v_{SL} + v_{SG}\,.$$

Once determined, the density of the mixture can be calculated using the equation

$$\rho_m = \rho_L H_L + \rho_G(1 - H_L)\,. \tag{6}$$

To determine the friction coefficient of a two-phase flow using the Moody curve [5], the Darcy–Weisbach equation for a single-phase flow, the value of the relative roughness of the pipe and the Reynolds number for a two-phase flow are used.

When calculating the Reynolds number for a two-phase flow, the gas-liquid mixture is considered as homogeneous over a limited interval:

$$\mathrm{Re}_f = \frac{v_m \rho_{mn} d}{\mu_m}\,. \tag{7}$$

μ_m determined from the equation,

$$\mu_m = \mu_L^{H_L} \mu_G^{(1-H_L)} \tag{8}$$

where μ_G is the gas viscosity.

Orkiszewski method

This method uses a combination of dependencies – Griffith and Wallis for the slug flow, Dance – Rosa for the transition flow and mist flow, and the dependence proposed by Orkiszewski for the slug flow.

The definition of the boundaries of two-phase flow regimes for the Orkiszewski method is shown in Table. 1.

Table 1

Flow mode	Conditions (mode boundary) Условия (граница режима)
Bubble	$\frac{v_{SG}}{v_m} < L_B$
Cork (Slug)	$\frac{v_{SG}}{v_m} > L_B$, $N_{GV} < L_S$
Transition	$L_M > N_{GV} > L_S$
Dispersed (Mist)	$N_{GV} > L_M$

The boundaries between bubble and plug regimes were defined by Griffith and Wallis, and for other regimes by Dance and Ross.

The boundaries between modes are determined from the following equations:

– between bubble and plug modes L_B:

$$L_B = 1{,}071 - 0{,}2218 \frac{v_m^2}{d}, \tag{9}$$

with restriction $L_B \geq 0{,}13$;

– between plug and transition modes L_S:

$$L_S = 50 + 36 N_{GV} \frac{q_L}{q_G}; \tag{10}$$

– between transition and dispersed modes L_M:

$$L_M = 75 + 84 \left(\frac{v_G q_L}{q_G} \right)^{0,75}. \tag{11}$$

In bubble mode H_L, it is determined from the equation

$$H_L = 1 - \frac{1}{2} \left[1 + \frac{v_m}{v_S} - \sqrt{\left(1 + \frac{v_m}{v_S}\right)^2 - \frac{4 v_{SG}}{v_S}} \right], \tag{12}$$

where v_m is the speed of the mixture; v_{SG} – reduced gas velocity; v_S – gas slip speed.

According to the Orkiszewski method, the average slip velocity v_S is assumed to be constant and equal to 0.8 ft/sec (0.24 m/s), the average flow density is found from equation (6), the hydrostatic pressure gradient is determined from

$$\left(\frac{dP}{dL}\right)_h = \rho_m g\cos\theta . \tag{13}$$

Friction pressure losses are determined from

$$\left(\frac{dP}{dL}\right)_f = \frac{f\rho_L\left(\frac{v_{SL}}{H_L}\right)^2}{2d} . \tag{14}$$

Friction coefficients are determined from the Moody diagram using relative roughness values and the Reynolds number for the fluid:

$$\mathrm{Re}_f = \frac{\rho_L\left(\frac{v_{SL}}{H_L}\right)d}{\mu_L} . \tag{15}$$

Slip density for plug mode:

$$\rho_m = \frac{\rho_L\left(v_{SL}+v_b\right)+\rho_G v_{SG}}{v_m+v_b} + \Gamma\rho_L , \tag{16}$$

where v_b is the rate of rise of the bubble; Γ – liquid distribution coefficient. The rate of bubble rise is defined as

$$v_b = C_1 C_2\sqrt{gd} . \tag{17}$$

The coefficients C_1 and C_2 are defined as a function of the Reynolds number for the bubble

$$\mathrm{Re}_B = \frac{\rho_L v_b d}{\mu_L} \tag{18}$$

and Reynolds numbers for liquid

$$\mathrm{Re} = \frac{\rho_L v_m d}{\mu_L} . \tag{19}$$

After calculating the Reynolds numbers for the bubble and for the liquid, the coefficients C_1 and C_2 can be determined from the graphs proposed by Griffith and Wallis [10].

Friction pressure loss is determined from the equation

$$\left(\frac{dp}{dL}\right)_f = \frac{f\rho_L v_m^2}{2d}\left[\left(\frac{v_{SL}+v_b}{v_m+v_b}\right)+\Gamma\right]. \tag{20}$$

The friction coefficient is determined from the Moody diagram at a given value of the Reynolds number for the fluid.

The liquid distribution coefficient Γ is determined from equations depending on the continuous liquid phase (oil or water) and the velocity of the gas-liquid mixture [7]. For the transition and dispersed modes, the Orkiszewski method uses the Dance and Ross dependencies [9].

Beggs–Brill method

Beggs and Brill proposed the following equation for the pressure gradient

$$\frac{dp}{dL}=\frac{\dfrac{f\rho_n v_m^2}{2d}+\rho_m g\sin\theta}{1-E_k}, \tag{21}$$

where E_k is the dimensionless pressure gradient of kinetic energy, defined as

$$E_k=\frac{v_m v_{SG}\rho_n}{p}. \tag{22}$$

The density of the mixture is calculated from the equation

$$\rho_m=\rho_L H_L(\theta)+\rho_G(1-H_L(\theta)). \tag{23}$$

The Beggs–Brill method takes into account the effect of pipe inclination θ on the value of H_L. Thus, slippage between moving phases H_L increases with increasing angle of inclination of the pipe from horizontal to vertical (upward flow), due to the force of gravity, which reduces the speed of the liquid. With a further increase in the angle of inclination, the liquid occupies the entire cross-section of the pipe and the slippage between phases decreases and decreases H_L.

Beggs and Brill proposed introducing a slope correction factor ψ:

$$\psi=\frac{H_L(\theta)}{H_L(0)}, \tag{24}$$

where $H_L(\theta)$ is the fluid retention at the angle of inclination θ from the horizontal; $H_L(0)$ – fluid retention with horizontal flow ($\theta=0$). $H_L(0)$ must be greater than or equal to the volume fraction of the no-slip fluid.

First you need to calculate $H_L(0)$:

$$H_L(0)=\frac{a\lambda_L b}{Fr^c}, \tag{25}$$

where Fr is the Froude number for a gas-liquid mixture,

$$Fr=\frac{v_m^2}{gd}; \tag{26}$$

a, b, c – empirical coefficients depending on the flow regime.

The slope correction factor ψ is determined as follows:

$$\psi=1+C\left[\sin(1{,}8\theta)-0{,}333\sin^3(1{,}8\theta)\right], \tag{27}$$

where θ is the actual angle of deviation of the pipe from the horizontal; C – fluid retention parameter ($C\geq 0$).

It is necessary to take into account that $C\geq 0$ ($C=0$ at $\psi=1$) and is determined from the dependence

$$C=(1-\lambda_L)\ln\left(e\lambda_L^f N_{LV}^g Fr^h\right), \tag{28}$$

where e, f, g, h are empirical coefficients depending on the flow mode and direction.

The friction coefficient of a two-phase flow is determined from equation (21). It is normalized by dividing by the no-slip coefficient of friction f_n, which is found from the Moody diagram, or by using the no-slip value of the Reynolds number and the equation for the friction coefficient of smooth pipes.

Beggs and Brill proposed the following equation to determine the coefficient of friction for two-phase flow:

$$\frac{f}{f_n} = e^S . \quad (29)$$

S determined from the expression

$$S = \frac{\ln y}{-0{,}0523 + 3{,}182 \ln y - 0{,}8725(\ln y)^2 + 0{,}01853(\ln y)^4}, \quad (30)$$

$$y = \frac{\lambda_L}{(H_L(\theta))^2} . \quad (31)$$

In order $y = 1$ for the dependence to describe a single-phase flow, Beggs and Brill proposed the formula $S = \ln(2{,}2y - 1{,}2)$, where $1 < y < 1{,}2$.

3. Results and discussion. The study was conducted on the performance indicators of 40 wells of the A-1 oil field.

The initial data were divided into groups according to the diameter of the pumping pipes and the gas factor (table 2).

Table 2

Parameters of oil field wells	Meaning
Tubing diameter (inch)	3,5 and 4,5
Specific gravity of liquid	0,85
Specific gravity of gas	0,7
Liquid flow rate (STB/D)	1088…6949
Gas factor (SCF/STB)	611…1898
Number of tests	40

Due to the lack of data from laboratory studies of changes in the physical properties of fluids for the studied range of pressures and temperatures and to predict them, the fixed oil model was used [11].

Calculations of changes in pressure loss using the methods under consideration were carried out for wells with different tubing diameters (3.5" and 4.5").

An analysis of the comparison between the calculated and actual values of pressure loss was carried out in terms of the percentage of error E_i, the average relative percentage of error E and the average deviation MD.

These indicators are determined as follows [12].

The error (%) is determined from the equation

$$E_i = 100 \frac{Y_c - Y_m}{Y_m},$$

where Y_c is the calculated value; Y_m – measured value.

The average relative error (%) is

$$E = \frac{\sum E_i}{n},$$

where n is the number of values.

The average deviation is determined from the equation

$$MD = \frac{|E - E_i|}{n}$$

The results of the analysis are presented in table. 3.

Table 3

Method	Analysis results for tubing diameter			
	3,5"		4,5"	
	E, %	MD	E, %	MD
Hagedorn-Brown	-6,7	16,9	-38,2	6,28
Beggs–Brill	20,7	36,46	-16,9	4,88
Orkiszewski	5,07	24,9	-14,8	4,57

Analysis of the data obtained showed that the Hagedorn-Brown method gives the best results for pipes with a diameter of 3.5", and the Orkiszewski method for 4.5".

The influence of the gas factor on the error of the calculated pressure loss values using the methods under consideration was also studied.

The field data were divided into two groups: the first - for wells with gas factor values of 611...938 SCF/STB, the second - 938...1898 SCF/STB. The results of the analysis are presented in table. 4.

Table 4

Method	Results of analysis at gas factor			
	611…938 SCF/STB		938…1898 SCF/STB	
	E, %	MD	E, %	MD
Hagedorn-Brown	-12,2	4,5	-26,1	5,5
Beggs–Brill	12,6	5,8	5,5	5,6
Orkiszewski	4,3	4,9	-8,1	4,9

The results obtained show that the Hagedorn-Brown method describes multiphase flow better than others for gas factor values from 611 to 938 SCF/STB, and the Orkiszewski method - from 938 to 1898 SCF/STB.

4. Conclusions. An analysis of methods for determining pressure losses in multiphase vertical flow (Hagedorn – Brown, Beggs – Brill and Orkiszewski) for various values of well operating parameters was carried out.

Based on the results obtained, it was established that determining pressure loss by the Hagedorn – Brown method gives the best results for smaller diameters of rising pipes (3.5") and at low values of the gas factor (611...938 SCF/STB), and the Orkiszewski method - for larger diameters lifting pipes (4.5") and for high gas factor values (938...1898 SCF/STB).

Conflict of interest.

The authors declare that they have no conflict of interest in relation to this research.

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